Solutions: Session 8

Exercise 1:

Answers:

a) In the system composed of a temperature-sensitive gage, the bridge voltage u_0 and the sensitivity S are:

$$u_o = u_i\left(\frac{R}{2R+\Delta R} - \frac{R}{2R}\right) = -u_i\left(\frac{1}{2} * \frac{\frac{\Delta R}{R}}{2+\frac{\Delta R}{R}}\right) = \frac{-u_i K\varepsilon}{2(2+K\varepsilon)} \approx \frac{-u_i K\varepsilon}{4}$$

$$S = \frac{u_o}{\varepsilon} \approx \frac{-Ku_i}{4}$$

It is worthwhile to note that to find the above expression, we have used the fact that $K\varepsilon \ll 1$. Indeed, typical values of temperature-induced strain close to room temperature (300 Kelvin) are of the order of 0.0001 (or 100 microstrain), and using the given expression for K, we obtain $K\varepsilon \sim 0.004$).

Now, to find an expression for sensitivity as a function of temperature, we consider:

$$\frac{dS}{dT} = \frac{-u_i}{4}.\frac{dK}{dT} = \frac{u_i}{4}.(200 * 0.005).e^{-0.005T} = \frac{Ku_i}{4} * 0.005 = -S * 0.005$$

Which, for small changes in temperature, ΔT, yields:

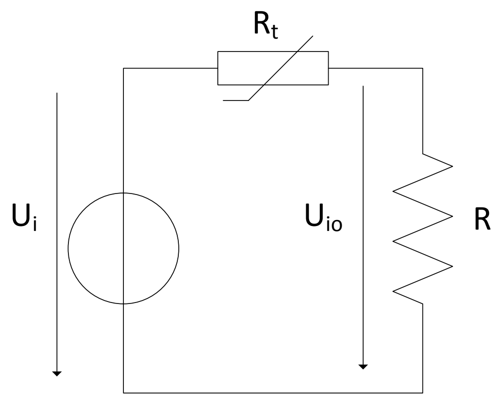
$$\frac{\Delta S}{S} = \frac{\Delta K}{K} = \boldsymbol{-0.005\Delta T}$$

b) The thermistor is added to compensate the *system's temperature dependence (it is important to understand that the thermistor will compensate the variation of the entire bridge, not only the arm with the gage!)*

When the thermistor R_t is added, the input voltage of the bridge becomes u_{io}, written as follows (refer to figure below) at constant room temperature:

$$u_{io} = \frac{R_{eq}}{R_{eq} + R_t} u_i$$

Where $R_{eq} = \frac{(R+\Delta R+R)*2R}{(R+\Delta R+R)+2R} = 2R * \frac{2+K\varepsilon}{4+K\varepsilon} \approx R$ (since $K\varepsilon \ll 1$, as explained in part (a)). Thus, for the remainder of this part, we shall **replace $\boldsymbol{R_{eq}}$ by $\boldsymbol{R}$**.



Thus, we obtain:

$$u_{io} = \frac{R}{R + R_t} u_i$$

Comparing with the expression for sensitivity obtained in part (a), the new sensitivity is given by:

$$S = \frac{u_o}{\varepsilon} \approx \frac{-K u_{io}}{4} = \frac{-u_i K}{4} * \frac{R}{R + R_t}$$

Consider a change dS in this sensitivity when the temperature changes by dT. This may be written as follows:

$$dS = \frac{-u_i}{4}\left[\frac{R}{R + R_t} dK - \frac{KR}{(R + R_t)^2} dR_t\right]$$

Let the temperature coefficient of the thermistor be given by $\alpha_t = \frac{1}{R_t} \cdot \frac{dR_t}{dT}$. Putting the expression for K in the above equation and simplifying, we obtain:

$$dS = \frac{-u_i K}{4} * \left(-\frac{\beta R}{R + R_t} dT - \frac{\alpha_t R_t R}{(R + R_t)^2} dT\right)$$

$$\frac{dS}{S} = -\frac{\beta R}{R + R_t} dT - \frac{\alpha_t R_t R}{(R + R_t)^2} dT$$

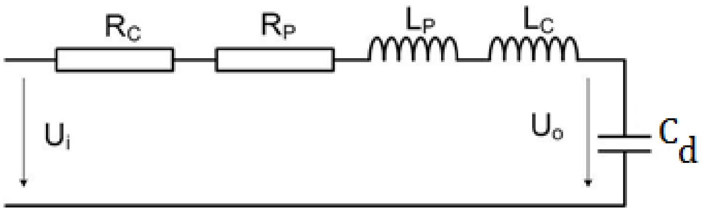
Where $\beta = 0.005$.

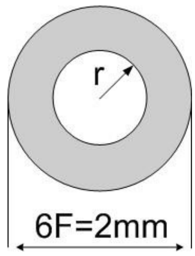
Now, proper temperature compensation implies that there should be no relative change in sensitivity with change in temperature, or $\frac{dS}{S} = 0$. For this condition to hold, we require that:

$$\boldsymbol{\alpha_t} = -\beta * \left(1 + \frac{R}{R_t}\right) = \mathbf{-0.005 * (1 + \frac{\boldsymbol{R}}{\boldsymbol{R_t}})}$$

Exercise 2:

Answers:





External diameter of catheter[1] = 6 F, or equivalently, 2 mm

a) In this scenario, the catheter has not been pinched yet, so R_P and L_P are absent. Thus, the transfer function (U_o/U_i) can be written as:

$$\frac{U_o}{U_i} = \frac{\frac{1}{j\omega C_d}}{R_C + j\omega L_C + \frac{1}{j\omega C_d}} = \frac{1}{1 - \omega^2 C_d L_C + j\omega C_d R_C}$$

Now, the transfer function for a general damped second-order system is given by:

$$H = \frac{K}{\left(\frac{j\omega}{\omega_n}\right)^2 + \left(\frac{2\xi j\omega}{\omega_n}\right) + 1}$$

We see that our transfer function is of the same form as the one shown above. Comparing both transfer functions, we obtain the following expressions for the natural frequency and damping ratio for the un-pinched catheter system:

$$\omega_n(natural\ frequency) = \frac{1}{\sqrt{C_d L_C}}$$

$$\xi(damping\ ratio) = \frac{R_c}{2}\sqrt{\frac{C_d}{L_C}}$$

b) It is given that the pinch reduces the diameter of the catheter by 75%. Thus, the radius of the pinched section is:

$$r_P = 0.25\, r$$

Let the length of the pinch be denoted by l_P and the viscosity of water by η. Then, the lumped-model equivalent resistances R_P and R_C of the pinched and non-pinched portions of the catheter can be respectively calculated from the expression for Poiseuille flow in a cylindrical tube as follows:

$$R_P = \frac{8\eta l_P}{\pi r_P^4} = \frac{8 * 0.001 * l_P}{\pi(0.46 * 10^{-3} * 0.25)^4} = 1456 * 10^{10} * l_P$$

[1] The units 'F' refer to the French size or the French gauge system for measuring catheter sizes. The French size = 3 times the diameter in millimetres. More information can be found here: https://en.wikipedia.org/wiki/French_catheter_scale

$$R_C = \frac{8\eta(l - l_P)}{\pi r^4} = 8 * 0.001 * \frac{l - l_P}{\pi(0.46 * 10^{-3})^4} = 5.69 * 10^{10} * (1 - l_P)$$

The lumped-model equivalent inductances of the pinched and un-pinched portions of the catheter, L_P and L_C respectively, can be calculated as the mass of water contained in that portion of the tube divided by the cross-sectional area in that section. This is done as follows:

$$L_P = \frac{m_P}{A_P^2} = \frac{\rho l_P \pi r_P^2}{(\pi r_P^2)^2} = \frac{\rho l_P}{\pi r_P^2} = \frac{10^3 l_P}{\pi(0.46 * 10^{-3} * 0.25)^2} = 24 * 10^9 * l_P$$

$$L_C = \frac{\rho(l - l_P)}{\pi r^2} = \frac{10^3(l - l_P)}{\pi(0.46 * 10^{-3})^2} = 1.5 * 10^9 * (1 - l_P)$$

The compliance of the sensor diaphragm is given as follows:

$$C_d = \frac{1}{E_d} = \frac{\Delta V}{\Delta P} = \frac{1}{0.49 * 10^{15}\,\frac{\mathrm{N}}{\mathrm{m}^5}}$$

And, the damping ratio ξ_P in the pinched condition is given by:

$$\xi_P = \frac{R_p + R_c}{2}\sqrt{\frac{C_d}{L_P + L_C}}$$

Inserting values, we obtain:

$$\xi_P = 0.4 = \frac{5.69 * 10^{10} + 1450 * 10^{10} * l_P}{2}\sqrt{\frac{1/(0.49 * 10^{15})}{1.5 * 10^9 + 22.5 * 10^9 * l_P}}$$

Solving the above quadratic equation and retaining the positive result (and rejecting the negative one):

$$\boldsymbol{l_p = 0.16\ \mathrm{m}}$$

c) Now, we know from the general form of a second-order system equation, that the natural undamped frequency, ω_n is given by $1/\sqrt{L_{C0}C_d}$, where L_{C0} is the catheter inductance and C_d is the sensor diaphragm compliance. We now calculate this natural undamped frequency for a pinched and un-pinched catheter as shown below.

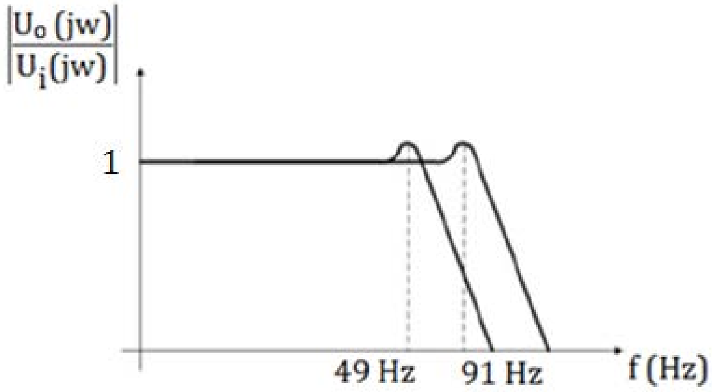
With pinch:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{(L_p + L_c)C_d}} = \frac{1}{2\pi}\sqrt{\frac{0.49 \times 10^{15}}{1.5 \times 10^9 + 22.5 \times 10^9 \times 0.16}} = 49\ Hz$$

Without pinch:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{L_{C0}C_d}} = \frac{1}{2\pi}\sqrt{\frac{0.49 \times 10^{15}}{1.5 \times 10^{9}}} = 91\ Hz$$

Thus, **the natural frequency decreases from 91 Hz (without pinch) to 49 Hz (with pinch)**. As a side-remark, we note that ξ_0, the damping ratio in the absence of a pinch, is 0.033. Thus, pinching the catheter increases its damping ratio (from 0.033 to 0.4).



d) As seen during the course on slide 41 in Chapter Resistive Sensors, as more harmonics (for the purpose of this discussion, we consider the natural frequency as the first harmonic, twice the natural frequency as the second harmonic, and so on) are added, the synthesized waveform resembles the original signal more closely. We see (in the same slide) that even after the addition of the first 6 harmonics, there is still some noticeable difference between the synthesized waveform b and the original signal a. Thus, the accuracy of measuring a signal, in this case the arterial blood pressure, highly depends on the number of harmonics that can be captured by the measuring system.

From the plot provided above, we observe that the pinched catheter system has a lower cutoff frequency of 49 Hz (compared to 91 Hz for the un-pinched catheter system). For humans, we will be able to capture 14 harmonics (calculated as 49 Hz/3.3 Hz), and for dogs, we will be able to capture 9 (calculated as 49 Hz/5 Hz). This is a sufficient number to capture the high-frequency details of the respective signals. However, for mice, we will only be able to capture 2 harmonics (49 Hz/22 Hz), and the pinched catheter will significantly distort this signal. Even with the un-pinched catheter, we will only be able to capture 4 harmonics (91 Hz/22 Hz), and even this signal might be distorted.